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An Area Computation Based Method for RAIM Holes Assessment

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Abstract. Receiver Autonomous Integrity Monitoring (RAIM) is a method implemented within the receiver to protect users against satellite navigation system failures. Research has shown that traditional methods for the determination of RAIM holes (i.e. places where less than five satellites are visible and available) based on spatial and temporal intervals (grids) compromise accuracy due to the constraint of computation load. Research by the authors of this paper has addressed this and developed a new algorithm to determine RAIM holes using bounded regions instead of approximation based on grid points.

This paper uses the new algorithm and proposes an area based method for the computation a RAIM satellite availability statistic based on the ratio of the total area of RAIM holes and the coverage area (regional or global area). Assessment over time is based on the interpolation using a model generated from snapshot spatial statistics at a relatively long temporal interval.

Test results show that the area-based method for the calculation of the RAIM satellite availability statistic is significantly more accurate with less computational load than the traditional grid points based approach.

Key words: Integrity monitoring, RAIM hole, GNSS, performance assessment

1 Background

Receiver Autonomous Integrity Monitoring (RAIM) is a receiver-based scheme to provide timely and valid warnings to users when a Global Navigation Satellite System (GNSS) is not able to meet the required navigation performance (RNP). RAIM algorithms are based on statistical consistency checks using redundant measurements. Consistency checks require five or more visible satellites, while in the case of Failure Detection and Exclusion (FDE), at least six visible satellites are required. A RAIM hole occurs when there are insufficient navigation satellites in view to provide an integrity check at a given location. It is defined as the area (or period) in which less than five GNSS satellites are in view above a mask angle of 7.5 degrees (Air Force Space Command, 1997). RAIM holes are the result of information shortage causing a RAIM algorithm to be unable to perform its function. Accordingly, an FDE hole can be defined as the area (time) in which less than six GNSS satellites are in view above a mask angle of 7.5 degrees.

The method commonly used is to overlay a grid over the area of interest and to search at the grid points (nodes) over time. The spatial and temporal intervals that have been used in RAIM availability assessment include: 5 degrees (spatial) and 5 minutes (temporal) (Ochieng et al., 2002), 3 degrees and 5 minutes (TSO-C129a,1996, RTCA/Do-229C, 2001) while Eurocontrol employs 0.25 degrees and 2.5 minutes in the AUGUR software (AUGUR). These sampling intervals are relatively large and are therefore susceptible to RAIM holes. Hence, if the spatial and temporal intervals are too large, some RAIM holes could pass undetected if they lie either between the spatial or temporal sampling points. However, smaller intervals result in a large number of sample points requiring increased computational resources. The grid-based search method is therefore always dependent upon a trade-off between accuracy and computational load.

A new method developed by Feng et al., (2006a) determines RAIM holes with a very high degree of accuracy. The key considerations in the new approach to determining RAIM holes are:

- The description of what constitutes a RAIM hole
- The determination of precise satellite coverage boundaries.
- The determination of the intersection points of satellites boundaries.

- Topological analysis of the regions formed by the intersection of these coverage boundaries

The coverage boundary of a satellite is normally considered to be a curve on the Earth surface. Any point on the curve has the same value of elevation angle as that of masking angle. A RAIM hole is a polygonal area on the Earth surface formed by the satellite coverage boundaries of less than five satellites in view. Since the Earth surface can be modelled by an ellipsoid, the area of a hole can be completely described by its borders, which is enclosed by segments of satellite coverage boundaries as demonstrated in figure 1. For each segment, there is a start point and an end point. These points are the intersections of coverage boundaries shared by the relevant two segments, e.g. in Figure 1 the intersection point A is both the start point of segment (AB) and the end point of segment (CA). The segments (AB, BC, CA) form a closed area.

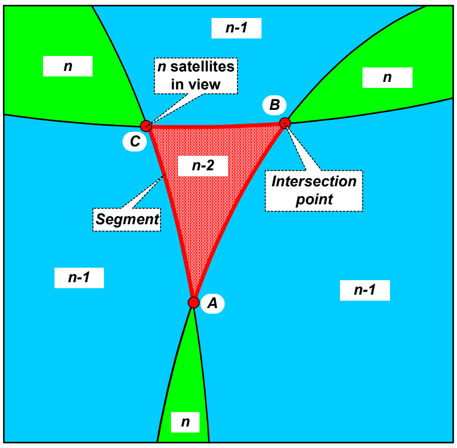

Figure 1. The boundaries intersection points and segments

The topology of the area formed by coverage boundaries is described briefly below:

If the boundaries of (m) satellites intersect at one point, there are ($2m$) regions around the intersection point. The intersection point has the maximum number (n) of visible satellites. The difference of the number of visible satellites between any two adjacent regions is 1. For m=2 or 3, there must exist at least one area where only (*n-2*) satellites are visible.

Based on the RAIM hole description and topology analysis, the determination of a RAIM hole is transformed into the determination of intersection points with six or less satellites in view and the adjacent segments of coverage boundaries. The determination of intersection points is extremely important because they define the start and end point of each segment. These points are referred to as crucial points in this paper. To determine the analytical solution of the segment between the start and end point is quite difficult in an ellipsoidal model. However, the segment can be described discretely with a number of points on the coverage boundary. The points on the segment between two crucial points pair (start, end) are referred to as critical points in this paper. Analysis shows that there exist at least three crucial points if a hole exists (Feng et al., 2006b).

The method here is to determine a polygonal area formed by the crucial and critical points of the segments on the coverage boundaries as shown in figure 2 (if $n \le 6$). This is a more accurate description of the area of a RAIM hole than grid-points based description as demonstrated in figure 3. The area with (*n-2*) satellites in view is about 6% of the total area while the grid point based method gives about 4.7%. The RAIM holes could be missed if the area is very small or the grid is not dense enough.

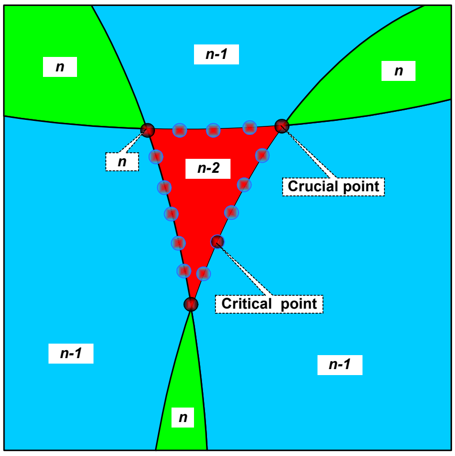

Figure 2. Key points of segments based method

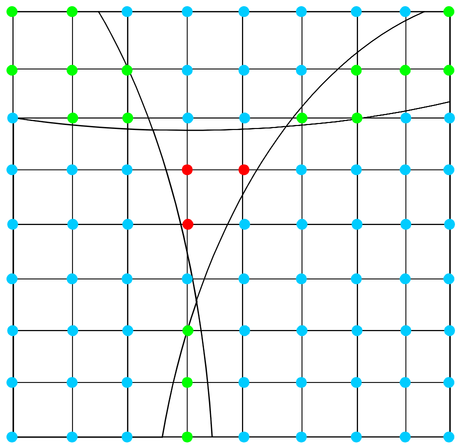

Figure 3. Grid points based area approximation

For a satellite of known position, there is a footpoint on the surface of the Earth, which is the intersection point of the Earth centre to satellite vector and the surface of the Earth as shown in figure 4. The position of a point on the coverage boundary has a deterministic relationship with the elevation angle, the position of the satellite, and the relative azimuth to the footpoint either using a spherical model or an ellipsoidal model of the Earth. However, it is not straightforward to determine the point using an ellipsoidal model. Two steps are normally used to determine the position of a boundary point on an ellipsoidal model. In the first step, the Earth is considered to be a sphere, and the point is assumed to be on the surface of the Earth. It is then straightforward to determine an approximate location of the point. In a second step, an iterative process is carried out to obtain the precise position of the point on an ellipsoidal model using the location in spherical model as the initial value. If the user is not on the surface of the Earth (e.g. an aircraft) and 3D location is known then the height information can be added in the second step to extend the coverage boundary from the surface of the Earth to any height of concern.

The process above is used to determine the intersection points. Initially, the intersection points are assumed to be on the surface of a spherical model of the Earth. The intersection points (As and Bs as shown in figure 4) of two boundaries can be calculated from the location of footpoints (F1, F2) and parameters derived from the masking angle using spherical trigonometry. In a second step, the intersection points (A_E, B_E) are calculated iteratively in the ellipsoidal model where local height information can also be considered. The accuracy of the location of these points depends on the number iterations. One iteration can reach an accuracy level of 1×10^{-4} degrees and six iterations can reach 1×10^{-10} degrees level (measured with respect to a fixed masking angle).

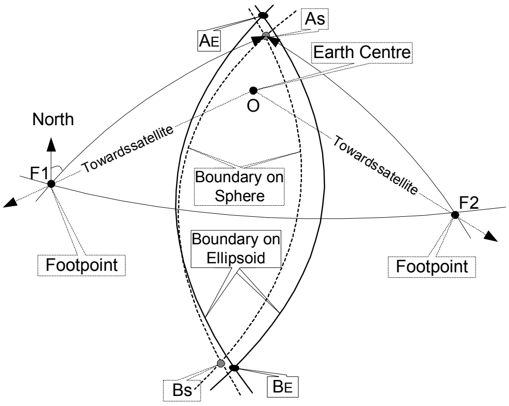

Figure 4. Intersection of two satellite coverage boundaries

Among these intersection points, only those with six or less satellites in view are considered to be crucial points and need to be identified. The position of the intersection point, the number of visible satellites, and related identities (e.g. PRN pair) of satellites whose coverage boundaries intersect at this point are recorded.

The existence of crucial points only indicates the existence a hole. There is no direct information about the size and the number of crucial points that encloses a hole. It is difficult to get a solution from the related identities of satellites that intersect at each crucial point. One reason is that the total number of crucial points could be very large and the hole could be formed by any number (more than two) of crucial points. The other reason is the existence of two points with the same identity since there are normally two intersection points for the converge boundaries of two satellites.

The next step is the determination of critical points at an azimuth interval along one coverage boundary starting from one crucial point and ending at next crucial point. The latter crucial point is taken as the start point for the other coverage boundary which intersects with the previous boundary. The process continues until a closed polygonal is found.

2 Area calculation and assessment of RAIM holes

According to the Radio Technical Commission for Aviation (RTCA), the grid points for current availability test are sampled every three degrees from zero to ninety degrees north (RTCA/Do-229C, 2001). The points on each latitude circle are separated evenly in longitude defined as:

$$long.interval = \frac{360}{ROUND(\frac{360}{\min(3\,degrees/\cos(latitude),\ 360)})} \qquad (1)$$

The longitude interval determined here enables uniformly distributed grid points in terms of area. Consequently, the area can be approximated by the number of points.

Obviously, the direct calculation of the area covered by RAIM holes is more accurate than point based approximations, and further enables a more accurate quantification of the RAIM satellite availability (Feng et al, 2005).

2.1 Area calculation of a polygon

The calculation of the polygon area of a RAIM hole on the surface of an ellipsoid is not straightforward. This is accomplished through an equal area projection from the ellipsoidal model to the spherical model. The projection involves two aspects, the determination of the authalic latitude, which transforms the latitude from the ellipsoidal model to the spherical model, and the determination of the radius of authalic sphere which has the same area as that in ellipsoidal model (Snyder, 1987; Maling, 1992).

The authalic latitude β, having the same surface area on a sphere as the ellipsoid, provides a sphere of equal area (authalic) to the ellipsoid:

$$\beta = \arcsin(q / q_p) \qquad (2)$$

where,

$$q=(1-e^2)\left(\frac{\sin\phi}{1-e^2\sin^2\phi}-\frac{1}{2e}\ln\left(\frac{1-e\sin\phi}{1+e\sin\phi}\right)\right) \tag{3}$$

$$q_p=1+\frac{1-e^2}{2e}\ln\left(\frac{1+e}{1-e}\right) \tag{4}$$

q_p is q evaluated for a ϕ of 90°, ϕ is the geodetic latitude. e denotes the first eccentricity of the ellipsoid.

The radius R_q of the sphere having the same surface area as the ellipsoid is calculated as follows:

$$R_q=a\sqrt{\frac{q_p}{2}} \tag{5}$$

where a is the semi-major axis of the ellipsoid.

The area of a RAIM hole region can be calculated using the numerical integration based on Green's Theorem for a polygonal area on a surface of a sphere.

2.2 Satellite availability assessment

The availability statistic based on the grid points method is to calculate the ratio of the number of available space-time points N_a versus the number of total sample points N_{total}, which can be expressed as:

$$A=\frac{N_a}{N_{total}} \tag{6}$$

where A is the availability.

The accuracy of the method is poor due to the approximation of an area by uniformly distributed points with a certain density. Therefore, it's always a trade-off between accuracy and the number of total grid points (density).

In contrast, the availability assessment method proposed here determines the exact areas of RAIM holes (unavailable region), and further calculates the ratio of the area of availability and the total area of concerned, which can be expressed as:

$$A=1-\frac{A_{Hole}}{A_c} \tag{7}$$

where, A is the availability, A_{Hole} is the area of RAIM holes, A_c is the area of the region concerned, or the surface of the Earth if a global assessment is performed.

Expression (7) enables a spatial determination of satellite availability at an instant in time. To carry out the availability assessment to cover a full geometry, a number of temporal samples (time domain) must be taken at a certain time interval. A relatively short time interval, e.g. one second inevitably results in a very large sample size. For example, for the grid-based method the product of the spatio-temporal assessment would be too large requiring very significant computational resources. This problem is negated by the area-based method which is able to use a relatively large time interval e.g. 50 seconds. Because of the accuracy of the area-based method and the predictability of RAIM holes, the performance in-between sample times can be interpolated by a model derived through a curve fitting process.

3 Results

A global RAIM holes calculation was carried out to verify the algorithm. The nominal constellation of 24 Global Positioning System (GPS) satellites (RTCA/Do-229C, 2001) was used. The Earth's ellipsoidal model WGS-84 was used.

The RAIM holes starting at 604000 to 605000 Seconds of Week (SoW) at a time interval of 200 seconds are shown in figures 5a to 5f. The solid lines are the boundaries of satellites footprint. The number of visible satellites at each intersection point is next to each point in the figure. Patches with different shapes and sizes are formed by the boundaries. The asterisks (*) are the crucial points and the dots are the critical points. The RAIM holes are the areas bounded by the asterisks and the dots, where only four or less satellites are in view. Figures 5a to 5f also show how the shapes and sizes of the RAIM holes change over time.

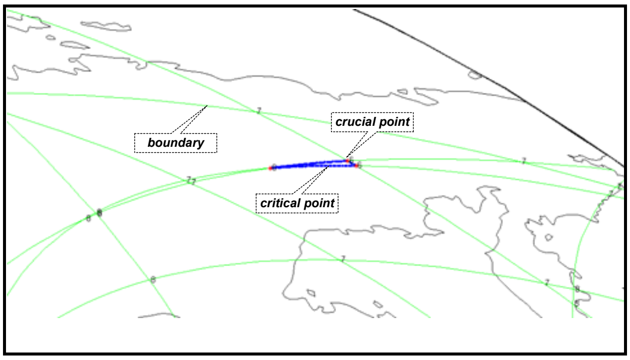

Figure 5a. RAIM hole at 604000 second

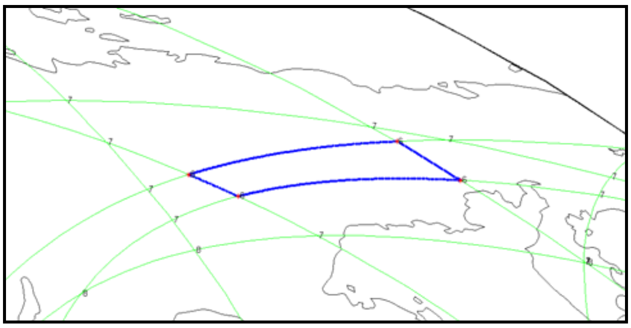

Figure 5b. RAIM hole at 604200 second

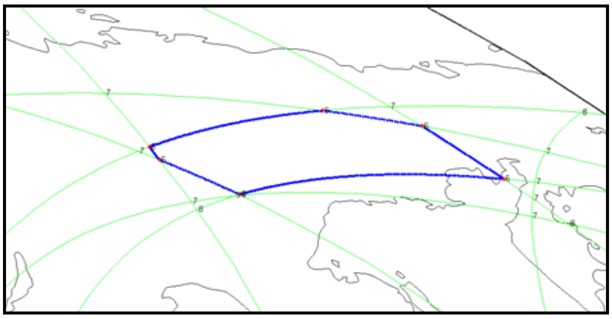

Figure 5c. RAIM hole at 604400 second

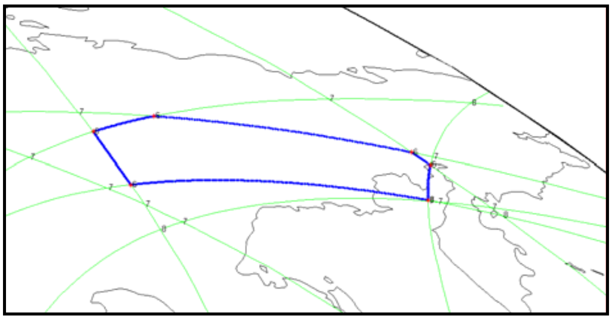

Figure 5d. RAIM hole at 604600 second

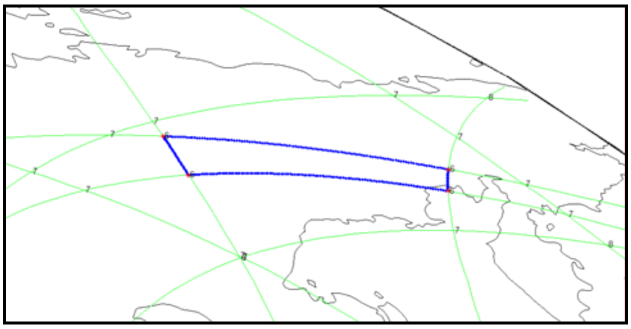

Figure 5e. RAIM hole at 604800 second

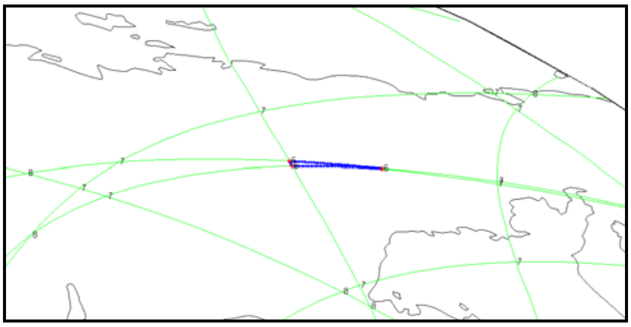

Figure 5f. RAIM hole at 605000 second

The areas of the RAIM holes and the global availability for a time interval of 100 seconds are listed in table 1.

In order to create a model to enable spatio-temporal determination of RAIM holes using a relatively large interval, a curve has been fitted to the data in table 1. This is shown in Figure 6. The asterisks (*) are the satellite availability at each sample time. Table 2 gives example errors between computed satellite availability and the corresponding interpolated availability. The very small errors in the interpolation confirm that a relatively long temporal interval can be used with the positive effect of significantly reducing the computational load.

Table1. RAIM hole area and availability

Time *SoW* (s)	RAIM hole Area (km^2)	Availability (%)	Figure
604000	7708.6	99.99849	5a
604100	128686.4	99.97483	
604200	292752.8	99.94273	5b
604300	493326.0	99.90350	
604400	700662.4	99.86294	5c
604500	805449.0	99.84244	
604600	699927.2	99.86308	5d
604700	492708.6	99.90362	
604800	292243.4	99.94283	5e
604900	128270.9	99.97491	
605000	7561.8	99.99852	5f

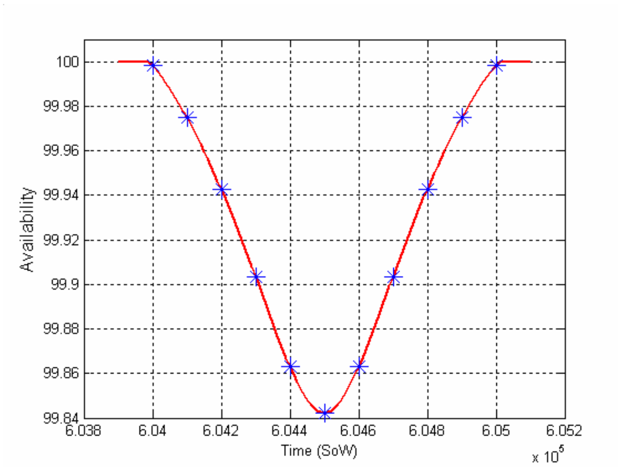

Figure 6. Curve fit of availability

Table2. Comparison of computed and interpolated availability

Time *SoW* (s)	Computed availability (%)	Interpolated availability (%)	Error (%)
604025	99.99419	99.99342	0.00077
604050	99.98818	99.98778	0.00040
604450	99.84799	99.84826	-0.00025
604550	99.84807	99.84835	-0.00028

Another example is the RAIM hole at 430950 SoW. The RAIM hole is small at only 1.6 square kilometres. The latitude and longitude of each of the three crucial points are:

$$(-63.30067970780234, -96.99308033516550)$$
$$(-63.29553056451819, -97.10230815708376)$$
$$(-63.30051390363663, -97.10833843315206)\,.$$

At 430950.49 SoW, the RAIM hole is smaller at only $36m^2$. The latitude and longitude of the three crucial points are:

$$(-63.29838672087586, -97.11264245143205)$$
$$(-63.29836181489549, -97.11316456619996)$$
$$(-63.29838572520419, -97.11319350372051)\,.$$

The area computation based method easily captures these RAIM holes, while for the grid based method a very dense grid would be required. In this example, a grid of less than 1×10^{-5} degree is required equivalent to more than 1.296×10^{15} sample points in order to capture this RAIM hole.

4 Conclusions

This paper has presented a new algorithm for the quantification of satellite availability with a particular focus of RAIM holes. Since the area of a RAIM hole changes in a deterministic way with the motion of the satellites whose boundaries form a region, a full geometry assessment (spatial and temporal) is possible via interpolation in the time domain using a relatively long temporal interval. This has the positive effect of delivering high accuracy with minimal computational load. The new approaches for determining RAIM holes and quantifying the RAIM satellite availability statistic in space and time, are significantly superior to the traditional grid-based method, both in terms of accuracy and computational load.

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