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VRS Virtual Observations Generation Algorithm

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Abstract. In the past few years, network RTK positioning technology, especially the VRS（virtual reference stations）technology, has been widely used in some parts of China and many countries of the world. In this paper, the authors mainly discuss the principle of VRS technology with corresponding formula deduction, and give detailed descriptions of VRS corrections and virtual observations generation algorithm as well as their applications.

Keywords. Network-RTK, VRS, corrections generation algorithm, virtual observation

1 Introduction

GPS virtual reference station positioning is the network RTK technology based on multiple reference stations. At least three reference stations are needed with this technology, the geodetic coordinates of the reference stations are precisely known. These reference stations continuously track the GPS satellites signal in order to determine the dispersive and non-dispersive biases. Those biases can be interpolated to the virtual reference station, so the rover around the VRS can take advantages of that information to get a better positioning accuracy. Meanwhile, the rovers have to send their approximate coordinates to a data process center (usually the master reference station), so the virtual observations can be generated at the approximate coordinate according to the approximate coordinates of rover and the corrections of the dispersive and non-dispersive biases. In fact, the VRS is established exactly at the approximate coordinate. Generally, the approximate coordinates are obtained from the pseudo range single positioning, whose accuracy can be control in meters or tens of meters, the baseline between the VRS and the rover is very short, that means the high coherence of some distance based biases between the rover and the VRS, so better result can be gotten from the traditional double-differential position method.

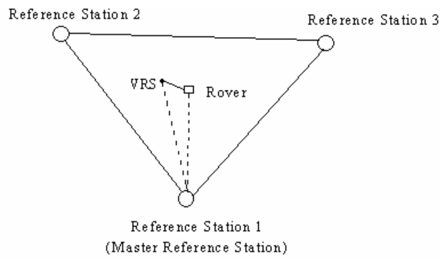

Fig 1.Virtual Reference Station

According to traditional relative positioning theory, the longer the baseline, the less the coherence of the distance based bias; It is a long-lasting contradiction in RTK application. The development of VRS technology has successfully solved that problem and extend the coverage of RTK service from 15 Km to at least 50 Km, this technology greatly improve the accuracy of the relative positioning in the middle and long baseline. Within the coverage of network-RTK service, even the single-frequency receiver can rapidly remove the ionospheric delay and resolve the integer ambiguities; CM-level positioning accuracy can be easily achieved, which is very encouraging. Besides, the platform of multiple reference stations should be fully utilized, some other services, for example GPS meteorology, deformation monitoring should also be available. Such departments as survey, meteorology, traffic, tourism, agriculture, and environment-protection will benefit a lot from this system. The focus of this paper will be on the virtual observations generation algorithm at the virtual reference station.

2 Biases Analysis

According to the GPS theory, the GPS positioning error can be divided into three parts:

- related to the GPS satellite: for instance the satellite clock bias. For one certain satellite, the satellite clock bias for the certain epoch is a constant, so it can be eliminated by differencing.
- related to the propagating path: this part comprise of ionospheric delay, tropospheric delay and multipath.

The tropospheric delay is non-dispersive, whose variations is relatively slowly compare to the ionospheric delay, in traditional GPS survey this bias can be diminished by applying a model correction. But in network RTK application, it was eliminated by the interpolated algorithm, whose data originate from the multiple reference stations. The algorithm will be discussed in the latter section. For the tropospheric delay is more sensitive to the altitude, the designers of the RTK network should consider such influence.

The ionospheric delay is dispersive, it is related to the frequency of the signal, and the ionospheric double-differenced corrections can be derived from pairs of reference stations by using the dual-frequency GPS receiver. It is clear that the ionosphere is more active, it vary quickly especially around equator regions, so refresh rate of the dispersive biases (e.g ionospheric delay) should be faster.

As for the multipath, it is difficult to diminish; the best way to avoid is to choose a better observation surrounding. Besides, using the antenna which can restrict the multipath is another choice. In VRS application, a advisable interpolated method can also diminish multipath to some extent.

3 related to the receiver: this part comprise of receiver clock bias and the noise. A high quality receiver can efficiently diminish this kind of error.

3 Double-differenced Corrections generation algorithm between multiple reference stations

For the convenience, we assume that all the multipath and the noise has already been controlled in a reasonable level, so is the orbit bias. Subscript a and b denotes the reference station, superscript m and n denotes satellites. According to the GPS theory, double-differenced observation equation can be described as:

$$\lambda(\nabla\Delta N_{a,b}^{m,n} + \nabla\Delta\varphi_{a,b}^{m,n}) = (\nabla\Delta\rho_{a,b}^{m,n} - \nabla\Delta I_{a,b}^{m,n} + \nabla\Delta T_{a,b}^{m,n}) \tag{1}$$

where $\nabla\Delta$ is the symbol of double-difference,

$\nabla\Delta\varphi_{a,b}^{m,n}$ is the double-differenced carrier phase observations,

$\nabla\Delta N_{a,b}^{m,n}$ is the double-differenced integer ambiguity,

λ is the wavelength of the signal,

$\nabla\Delta\rho_{a,b}^{m,n}$ is the double-differenced geometrical distance between the satellite and the phase center of receiver antenna,

$\nabla\Delta I_{a,b}^{m,n}$ is the double-differenced ionospheric delay,

$\nabla\Delta T_{a,b}^{m,n}$ is the double-differenced tropospheric delay.

In the equation above, $\nabla\Delta\varphi_{a,b}^{m,n}$ can be calculated from the phase observations. The coordinates of the reference stations is precisely known, the coordinates of the satellites can be obtained from the GPS ephemeris, so the term $\nabla\Delta\rho_{a,b}^{m,n}$ is also precisely known. After solving out the the double-differenced integer ambiguity $\nabla\Delta N_{a,b}^{m,n}$, the double-differenced ionospheric delay $\nabla\Delta I_{a,b}^{m,n}$ and the double-differenced tropospheric delay $\nabla\Delta T_{a,b}^{m,n}$ can eventually be obtained. It is clear that the key produce is to correctly determine the double-differenced integer ambiguity $\nabla\Delta N_{a,b}^{m,n}$ .In network RTK, reference stations part tens of kilometers from each other, distance-depended bias notably increases, and the ionospheric delay will have the most powerful influence on the integer ambiguity solution. Fortunately, many scholars have been working on this issue and come up with many operational techniques. Detailed information please refers to the other references; we would not discuss it in this paper.

The double-differenced ionospheric delay $\nabla\Delta I_{a,b}^{m,n}$ and the double-differenced tropospheric delay $\nabla\Delta T_{a,b}^{m,n}$ can be detached after the double-differenced integer ambiguity $\nabla\Delta N_{a,b}^{m,n}$ having been solved out. The dispersive biases are related to the frequency, so it is not difficult to detach them.

$$V = V^{disp} + V^{non-disp} \tag{2}$$

Moving the double-differenced ionospheric delay $\nabla\Delta I_{a,b}^{m,n}$ and the double-differenced tropospheric delay $\nabla\Delta T_{a,b}^{m,n}$ to the left of the equal mark in equation 1, they are the corrections V in the equation 3, so:

$$\begin{aligned} V &= \nabla\Delta T_{a,b}^{m,n} + (-\nabla\Delta I_{a,b}^{m,n}) \\ &= (\nabla\Delta\varphi_{a,b}^{m,n} + \nabla\Delta N_{a,b}^{m,n})\lambda - \nabla\Delta\rho_{a,b}^{m,n} \end{aligned} \tag{3}$$

Moreover, the expression of the ionospheric delay determined by dual-frequency GPS receiver can be described as:

$$I = \frac{f_2^2}{f_1^2 - f_2^2}[(\varphi_1\lambda_1 - \varphi_2\lambda_2) + (N_1\lambda_1 - N_2\lambda_2)] \quad (4)$$

From equation (4) we can deduce the expression of the double-differenced ionospheric delay $\nabla\Delta I_{a,b}^{m,n}$ between station a, b and satellites m, n.

$$\begin{aligned}\nabla\Delta I_{a,b}^{m,n} &= \frac{f_2^2}{f_1^2 - f_2^2}[(\nabla\Delta\varphi_{a,b,1}^{m,n}\lambda_1 - \nabla\Delta\varphi_{a,b,2}^{m,n}\lambda_2) \\ &+(\nabla\Delta N_{a,b,1}^{m,n}\lambda_1 - \nabla\Delta N_{a,b,2}^{m,n}\lambda_2)]\end{aligned} \quad (5)$$

With equation (2) and equation (5), the dispersive and non-dispersive biases can be detached.

$$V^{non-disp} = V - V^{disp} = V + \nabla\Delta I_{a,b}^{m,n} \quad (6)$$

In VRS application, a master reference station is to be chosen, all the double-differenced corrections refer to the baselines from the master station to each of the reference stations. Besides, the double-differenced corrections at VRS are also referred to the corrections at the baseline from the master to VRS. we assume that in fig 1 the reference station 1 as the master, each reference station have been tracing satellite m,n,so all the corrections can be summarized as $\nabla\Delta I_{1,2}^{m,n}$, $\nabla\Delta I_{1,3}^{m,n}$, $\nabla\Delta T_{1,2}^{m,n}$, $\nabla\Delta T_{1,3}^{m,n}$ (neglecting other biases).

4 VRS observations generation

In this step, we utilize the observations in the master reference stations, applying a geometric displacement, correcting the observations from the master to the VRS. The so-called geometric displacement is the geometric distance difference from a certain satellite to the master and the VRS.

The coordinates of the master reference station is precisely known, the coordinates of the satellites can be obtained from the GPS ephemeris, so the geometric distance between satellite and the phase center of receiver antenna can be also precisely determined. ρ_r^s denotes the geometric distance between the satellite and the receiver, it is obviously that:

$$\rho_r^s = \left[\left(\overline{X}_r - \overline{X}^S\right)^T\left(\overline{X}_r - \overline{X}^S\right)\right]^{\frac{1}{2}} \quad (7)$$

The position of VRS is provided by the pseudorange single positioning, it is precisely know

$$\rho_v^s = \left[\left(\overline{X}_v - \overline{X}^S\right)^T\left(\overline{X}_v - \overline{X}^S\right)\right]^{\frac{1}{2}} \quad (8)$$

The expression of geometric displacement can be described as:

$$\Delta\rho_{r,v}^s = \rho_v^s - \rho_r^s \quad (9)$$

The formula above can be used for place displacement.

The single-differenced observation equation between satellite m, n in a certain epoch can be written as:

$$\Delta\varphi_r^{m,n} = \frac{1}{\lambda}(\Delta\rho_r^{m,n} - \Delta I_r^{m,n} + \Delta T_r^{m,n}) - \Delta N_r^{m,n} \quad (10)$$

It is similar in the same epoch at the VRS.

$$\begin{aligned}\Delta\varphi_v^{m,n} &= \frac{1}{\lambda}(\Delta\rho_v^{m,n} - \Delta I_v^{m,n} + \Delta T_v^{m,n}) - \Delta N_v^{m,n} \\ &= \frac{1}{\lambda}(\Delta\rho_r^{m,n} + \nabla\Delta\rho_{r,v}^{m,n} - \Delta I_v^{m,n} + \Delta T_v^{m,n}) - \Delta N_v^{m,n}\end{aligned} \quad (11)$$

We can infer from the formula10 and 11 that the double-differenced observation equation between the master and the VRS can be concluded as:

$$\begin{aligned}&\nabla\Delta\varphi_{r,v}^{m,n} = (\Delta\varphi_v^{m,n} - \Delta\varphi_r^{m,n}) = \\ &\frac{1}{\lambda}(\nabla\Delta\rho_{r,v}^{m,n} - \nabla\Delta I_{r,v}^{m,n} + \nabla\Delta T_{r,v}^{m,n}) - \nabla\Delta N_{r,v}^{m,n}\end{aligned} \quad (12)$$

$\nabla\Delta\rho_{r,v}^{m,n}$ is the double-differenced geometric relationship, and can be described as:

$$\nabla\Delta\rho_{r,v}^{m,n} = (\rho_v^n - \rho_r^n) - (\rho_v^m - \rho_r^m) \quad (13)$$

In the formula (12), The single-differenced phase observation $\Delta\varphi_v^{m,n}$ is unknown and it is going to be determined, $\Delta\varphi_r^{m,n}$ is the single-differenced phase observation at the master reference station, and the double-differenced place displacement $\nabla\Delta\rho_{r,v}^{m,n}$ can be derived from equation (13), the double-differenced ionospheric delay $\nabla\Delta I_{r,v}^{m,n}$ and the double-differenced tropospheric delay $\nabla\Delta T_{r,v}^{m,n}$ can be obtained from the interpolation algorithm which we will discuss in next section. Once the double-differenced ambiguity $\nabla\Delta N_{r,v}^{m,n}$ is solved, the single-differenced phase observation $\Delta\varphi_v^{m,n}$ can be calculated.

5 The interpolation algorithm of double-differenced corrections

How to interpolate the corrections generated at the reference to virtual reference stations is another key issue in network RTK. Over the pass few years, many scholars have been working on it and have developed some efficient interpolation algorithms (Gao et al., 2002; Rizos et al., 2002; Dai et al., 2001; Fotopoulos and Cannon, 2001; Raquet and Lachapelle, 2001; Vollath et al., 2000). The main methods includes Linear Combination Model (LCM), Linear Intrplotation Method (LIM), Low-order Surface Model (LSM), and the Least-squares Collocation(LSC).

5.1 Linear Combination Model

According to formula (3), we assume that there are n reference stations, the double-differenced corrections between reference i and the master reference n can be depicted as:

$$V = \nabla\Delta T_{i,n} + (-\nabla\Delta I_{i,n}) = (\nabla\Delta\varphi_{i,n} + \nabla\Delta N_{i,n})\lambda - \nabla\Delta\rho_{i,n} \qquad (14)$$

Then, the corrections between the master and other reference station can be written as:

$$V_{u,n} = \alpha_1 V_{1,n} + \alpha_2 V_{2,n} + \alpha_3 V_{3,n} + \cdots + \alpha_{n-1} V_{n-1,n} \qquad (15)$$

It is obviously that the correction between the VRS and the master is the linear combination of reference stations i and the master station n, the parameter α is going to be resolve, and they suffice the following conditions.

$$\sum_{i=1}^{n} \alpha_i = 1 \qquad (16)$$

$$\sum_{i=1}^{n} \alpha_i (\overrightarrow{X}_u - \overrightarrow{X}_i) = 0 \qquad (17)$$

$\overrightarrow{X}_u$ and $\overrightarrow{X}_i$ denotes the horizontal coordinates vector for the user station and the reference i respectively.

$$\sum_{i=1}^{n} \alpha_i^2 = Min \qquad (18)$$

In form of matrix, formula (16) and (17) can be described as:

$$\begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ \Delta X_{1,n} & \Delta X_{2,n} & \cdots & \Delta X_{n-1,n} & 0 \\ \Delta Y_{1,n} & \Delta Y_{2,n} & \cdots & \Delta Y_{n-1,n} & 0 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} = \begin{pmatrix} 1 \\ \Delta X_{un} \\ \Delta Y_{un} \end{pmatrix} \qquad (19)$$

Matrix B is

$$B = \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ \Delta X_{1,n} & \Delta X_{2,n} & \cdots & \Delta X_{n-1,n} & 0 \\ \Delta Y_{1,n} & \Delta Y_{2,n} & \cdots & \Delta Y_{n-1,n} & 0 \end{pmatrix},$$

$$\vec{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}, \quad C = \begin{pmatrix} 1 \\ \Delta X_{un} \\ \Delta Y_{un} \end{pmatrix}.$$

And it is not difficult to resolve $\vec{\alpha}$.

$$\vec{\alpha} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} = B^T (BB^T)^{-1} C \qquad (20)$$

From the formula above we can find that the linear combination model needs at least three reference stations, and the correction $\vec{\alpha}$ is correlated to B and C, that means it is correlated to structure of the multiple reference network and the location of the user. When the user moves, the coefficient $\vec{\alpha}$ changes and should be recalculated. If the user is at a mobile vehicle, the burden of the VRS communication system enormously increased, which is the main drawback of this model.

5.2 Linear Interpolation Method

This method is suggested to derive the differential ionosphereic bias from the reference stations around the user. Then this method is extended to derive other biases such as tropospheric delays and orbit error. The corrections from the master to VRS can be described as:

$$V_{u,n} = (\Delta X_{u,n} \quad \Delta Y_{u,n}) \begin{pmatrix} a \\ b \end{pmatrix} \qquad (21)$$

$\Delta X_{u,n}$, $\Delta Y_{u,n}$ is the coordinates difference from the VRS to the master reference station n .Once the position of VRS is confirmed, they are also precisely known. So the key of this method is to resolve the coefficient a, b among the reference station, the interpolation model can be described as:

$$\begin{pmatrix} V_{1,n} \\ V_{2,n} \\ \vdots \\ V_{n-1,n} \end{pmatrix} = \begin{pmatrix} \Delta X_{1,n} & \Delta Y_{1,n} \\ \Delta X_{2,n} & \Delta Y_{2,n} \\ \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \qquad (22)$$

$\Delta X_{i,n}$ 、 $\Delta Y_{i,n}$ is the coordinates difference from the reference station i to the master station n. Linear Interpolation Method needs at least 3 reference stations, if the number of reference station is more than three, coefficient a, b can be determined by least-square adjustment.

$$\begin{pmatrix} a \\ b \end{pmatrix} = (A^T A)^{-1} A^T V ,$$

$$A = \begin{pmatrix} \Delta X_{1,n} & \Delta Y_{1,n} \\ \Delta X_{2,n} & \Delta Y_{2,n} \\ \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} \end{pmatrix}, \quad V = \begin{pmatrix} V_{1,n} \\ V_{2,n} \\ \vdots \\ V_{n-1,n} \end{pmatrix}.$$

Now, the corrections can be gotten from equation (21).

The coefficient a, b is the function of different biases in the Linear Interpolation Method, but in the Linear Combination Model, the coefficient is just the function of coordinates difference, which do not include the information of biases, that is the main difference between these two method. Besides, we can prove that when the number of the reference stations is 3, these two methods are equivalent.

5.3 low-order surface model

It is obviously that this method means to use a low-order surface to fit the spatial-related biases between multiple reference stations. The order of the fitting function could one or two, the variables of the function can be two (horizontal coordinates) or three (horizontal and altitude coordinates). Considering the altitude, the one order fitting function can be described as:

$$V = (\Delta X \quad \Delta Y \quad \Delta H \quad 1) \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \tag{23}$$

We need at least five reference stations to determine the coefficients a, b, c, d, if the number of reference station is more than five, we can get a least-square solution. We assume $\Delta X_{i,n}$ and $\Delta Y_{i,n}$ are the coordinate difference from the reference station i to the master reference station n, $\Delta H_{i,n}$ is the altitude difference between the reference station i and the master reference station n.

$$V = \begin{pmatrix} V_{1,n} \\ V_{2,n} \\ \vdots \\ V_{n-1,n} \end{pmatrix}$$

$$A = \begin{pmatrix} \Delta X_{1,n} & \Delta Y_{1,n} & \Delta H_{1,n} & 1 \\ \Delta X_{2,n} & \Delta Y_{2,n} & \Delta H_{2,n} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \Delta X_{n-1,n} & \Delta Y_{n-1,n} & \Delta H_{n-1,n} & 1 \end{pmatrix}$$

So the solution of equation (23) can be written as:

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = (A^T A)^{-1} A^T V \tag{24}$$

After the coefficients a, b, c, d being determined, we can get the corrections from following formula.

$$V_{u,n} = (\Delta X_{u,n} \quad \Delta Y_{u,n} \quad \Delta H_{u,n} \quad 1) \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \tag{25}$$

If neglecting the influence of altitude, formula (23) can be written as

$$V = (\Delta X \quad \Delta Y \quad 1) \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{26}$$

It can be used to estimate the ionospheric delay. Besides the methods above, other interpolation algorithm please refer to relative reference.

6. Locating the user

We assume that the user are also tracking satellite m, n in the same epoch, the single-differenced observation equation can be expressed as:

$$\Delta\varphi_u^{m,n} = \frac{1}{\lambda}(\Delta\rho_u^{m,n} - \Delta I_u^{m,n} + \Delta T_u^{m,n}) - \Delta N_u^{m,n} \tag{27}$$

Meanwhile, the double-differenced observation equation between the user and VRS can be described as:

$$\begin{aligned} \Delta\varphi_u^{m,n} - \Delta\varphi_v^{m,n} &= \frac{1}{\lambda}[(\nabla\Delta\rho_{v,u}^{m,n}) - (\Delta I_u^{m,n} - \Delta I_v^{m,n}) \\ &+ (\Delta T_u^{m,n} - \Delta T_v^{m,n})] - \nabla\Delta N_{v,u}^{m,n} \end{aligned} \tag{28}$$

For the user is adjacent to VRS, the biases cause by atmosphere are approximately the same, so

$$\Delta I_u^{m,n} - \Delta I_v^{m,n} \approx 0$$
$$\Delta T_u^{m,n} - \Delta T_v^{m,n} \approx 0$$

Equation 28 can de written as:

$$\Delta\varphi_u^{m,n} - \Delta\varphi_v^{m,n} = \frac{1}{\lambda}\nabla\Delta\rho_{v,u}^{m,n} - \nabla\Delta N_{v,u}^{m,n} \tag{29}$$

Now, we can get the position of the user according to the principle of double-differenced relative positioning.

7. Conclusion

This paper mainly discusses the principle and algorithm of using multiple reference stations to generate the observations at VRS. In the past a few years, multiple reference stations based RTK positioning technology has been widely used in some parts of China and many countries of the world, which has bring lots of benefits to the users. The accuracy and precision of GPS positioning has been tremendously improved. The users do not even to set up a reference station and they can get cm-level position accuracy even just with a single-frequency receiver.

The result of this paper has a guidance meaning for the network-RTK application. Besides, the continuously running reference stations would become the infrastructure which would provide more services for different customers. With the development of 3G communication system and the wireless access technology, the corrections parameters can be distributed with a relative low cast. Integration with the internet application may be the future direction that Network-RTK takes.

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